DUPLICAZIONE IV

Le formule di addizione possono essere utilizzate per ricavare quelle di duplicazione. Vediamo come.

Vogliamo calcolare $\operatorname{sen} 2\alpha$ e $\cos 2\alpha$.

Sappiamo che: $\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$

$\operatorname{sen} 2\alpha$ lo possiamo scrivere come

$\boxed{\operatorname{sen} 2\alpha = \operatorname{sen}(\alpha+\alpha)}$ applichiamo ora le formule precedenti

$\operatorname{sen} 2\alpha = \operatorname{sen}(\alpha+\alpha) = \operatorname{sen}\alpha\cos\alpha + \cos\alpha\operatorname{sen}\alpha = 2\operatorname{sen}\alpha\cos\alpha$

quindi

$$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha \qquad (1)$$

Ricaviamo nello stesso modo $\cos 2\alpha$.

$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \operatorname{sen}\alpha\cdot\operatorname{sen}\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha.$$

quindi

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha \qquad (2)$$

Esempio 1)

Sapendo che $0° < \alpha < 90°$ e $\operatorname{sen}\alpha = \frac{3}{5}$ calcolare $\operatorname{sen} 2\alpha$, $\cos 2\alpha$, $\operatorname{tg} 2\alpha$, $\operatorname{cotg} 2\alpha$, $\sec 2\alpha$, $\operatorname{cosec} 2\alpha$.

$$0° < \alpha < 90° \quad \Rightarrow \quad 0° < 2\alpha < 180°$$

$$\operatorname{sen}\alpha = \frac{3}{5} \quad \Rightarrow \quad \cos\alpha = +\sqrt{1-\frac{9}{25}} = +\sqrt{\frac{25-9}{25}} = +\frac{4}{5}$$

Applico (1)

$$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$$

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\operatorname{tg} 2\alpha = \frac{\operatorname{sen} 2\alpha}{\cos 2\alpha} = \frac{24}{\cancel{25}}\cdot\frac{\cancel{25}}{7} = \frac{24}{7} \qquad \operatorname{cotg} 2\alpha = \frac{7}{24}$$

$\sec 2\alpha = \frac{1}{\cos 2\alpha} = \frac{25}{7}$ $\operatorname{cosec} 2\alpha = \frac{1}{\operatorname{sen} 2\alpha} = \frac{25}{24}$

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$$\frac{1-\cos 2\alpha}{1+\cos 2\alpha} \cdot \cot g\,\alpha =$$

$$\frac{1-(\cos^2\alpha - \operatorname{sen}^2\alpha)}{1+\cos^2\alpha - \operatorname{sen}^2\alpha} \cdot \left(\frac{\cos\alpha}{\operatorname{sen}\alpha}\right) =$$

$$\frac{\underbrace{1-\cos^2\alpha}_{\operatorname{sen}^2\alpha} + \operatorname{sen}^2\alpha}{\underbrace{(1-\operatorname{sen}^2\alpha)}_{\cos^2\alpha} + \cos^2\alpha} \cdot \left(\frac{\cos\alpha}{\operatorname{sen}\alpha}\right) =$$

$$\frac{\cancel{2}\operatorname{sen}^2\alpha}{\cancel{2}\cos^2\alpha} \cdot \left(\frac{\cancel{\cos\alpha}}{\cancel{\operatorname{sen}\alpha}}\right) = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \boxed{tg\,\alpha}$$

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$$\cos 2\alpha - \frac{\cos\alpha \cdot \operatorname{sen} 2\alpha}{\operatorname{sen}\alpha} =$$

$$\cos^2\alpha - \operatorname{sen}^2\alpha - \frac{\cos\alpha(2\operatorname{sen}\alpha\cos\alpha)}{\operatorname{sen}\alpha} =$$

$$\cos^2\alpha - \operatorname{sen}^2\alpha - \frac{2\,\cancel{\operatorname{sen}\alpha}\cos^2\alpha}{\cancel{\operatorname{sen}\alpha}} =$$

$$\cos^2\alpha - \operatorname{sen}^2\alpha - 2\cos^2\alpha =$$

$$-\cos^2 - \operatorname{sen}^2\alpha = -1\,(\underbrace{\cos^2 + \operatorname{sen}^2\alpha}_{=1}) = -1(1) = \boxed{-1}$$

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Per 27 Marzo